Bias and MSE exercises

General instructions. For each of these case studies, generate approximately 10 data sets that meet the criterion. Then use the results from those 10 analyses to calculate the bias, average standard error and mean square error.

1. *Efficiency of Mean, Median and Trimmed Mean.* Generate 10 samples of size 10, 40, and 90 from a standard normal distribution. Look at the average bias of the mean, median and 10% trimmed mean for each of those sample sizes.
2. *Bias correction of the variance estimates.* Generate 10 samples of size 10, 40, and 90 from a standard normal distribution. Calculate the variance using both the sum-of-squares/*n* and sum-of-squares/(*n-1)* formulae. Show that the *n-1* formula is unbiased.
3. *Robustness of Mean, Median and Trimmed Mean.* You can make an epsilon contaminated distribution, by the R expression ifelse(runif(n)<epsilon,rnorm(n),rnorm(n)\*3). Calculate the bias, m.s.e., and average s.e., for the Mean, Median and 10% trimmed mean for epsilon of .05, .15 and .25 (use 10 samples from each distribution).
4. *Bayesian estimate: Binomial distribution.*  The MLE for the binomial distribution is *X/n,* while the Bayesian estimate with a Beta(*a,b)* prior is *(X+a)/(n+a+b).* (The standard errors can be calculated using the s.d. of the beta distribution). Generate ten data sets for *p=.5,* and *n=4,9,16,25,100.* Look at the bias and m.s.e. for both the MLE and the Bayesian estimate starting from a Beta(2,2) prior.
5. *Bayesian estimate: informative prior.* Same as number 4, except now let *p=.25.*
6. *Bayesian estimate: normal distribution.* Draw samples of size 4, 9, 16, 25 and 100 from a standard normal distribution. Calculate the bias and m.s.e. for the sample mean, a Bayesian estimate using a N(.1,1) prior and a Bayesian estimate using a N(1,1) prior.
7. *Bayesian estimate:* normal distribution. Same as problem 6, only use a N(.5,1) and N(.5,5) priors.